Roadmap Spanners

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Why Roadmaps and Roadmap Spanners?

Produce sparse graphical representations that:
– reflect the connectivity of the configuration space
– and can be used to efficiently answer online queries with good quality paths

• Posed as an important challenge for motion planning [Agarwal, ‘11]

• Good for resource constrained robots, potentially interfacing with a computing cloud

• Useful in higher-dimensional challenges such as mobile manipulation:
  – Roadmaps can store experience! They are path libraries!

From the work on “Dynamic Roadmaps” by Kallmann, Mataric
NEAR-

OPTIMAL

MOTION
Roadmaps and Path Quality

- A fully connected graph gives asymptotically optimal solutions
  - Resembles exhaustive search, results quickly in a huge data structure
- Connecting to $k$ closest neighbors is efficient [PRM, Kavraki et al. ’96]
  - Doesn’t result in an asymptotically optimal solution for constant $k$

From percolation theory
It is sufficient if we attempt to connect any new sample with approximately $k = \log n$ neighbors, where $n$ is the number of nodes in the roadmap.

[kPRM* - Karaman, Frazzoli ‘11]

- Efficiency challenge
  - Asymptotically optimal roadmaps are large and dense
Asymptotic Near-Optimality & Graph Spanners

- A $t$-spanner is a sparse subgraph
- For every shortest path in the original graph
  - There is a path in the spanner that is no longer than $t$ times the original length

\begin{itemize}
  \item Compute $k$-PRM*
  \item Return its spanner
\end{itemize}

\[\text{Potential new edge length} = 1.0\]
\[\text{Existing shortest path length} = 1.5\]

- Giving rise to a sequential approach:
  - Compute $k$-PRM*
  - Return its spanner

\[\text{Marble, Bekris IROS '11}\]
\[\text{Based on the graph spanner approach by Baswana, Sen '07}\]
Incremental Roadmap Spanner

- Start with the asymptotically optimal k-PRM*
- Interleave an incremental spanner algorithm
- Result: An asymptotically near-optimal planner
  - Smaller average increase in path length than the stretch factor
  - Sparse roadmap with smaller memory footprint
  - Faster construction and online query resolution

[Marble, Bekris ISRR ’11, IEEE Transactions on Robotics ’13]
Sparse Roadmap Spanner (SPARS)

• Up to this point: Solutions add all samples in the roadmap
• Idea: Asymptotic Near-Optimality with Additive Cost
  [Dobson, Krontiris, Bekris WAFR ’12, IJRR ’13 (accepted)]
• Consider two graphs in parallel:
  - Dense Graph: Asymptotically Optimal ($\delta$-PRM*)
  - Roadmap Spanner: Asympt. Near-Optimal, Not all nodes added (!)

• When should samples be added to $S$?
  - If necessary for coverage, connectivity, optimality
• When should the sampling stop?
  - Criterion: After $M$ consecutive failures to add a node

Similar objectives in recent work by Salzman, Agarwal, Halperin, Shaharabani ICRA ‘13
SPARS: Node Selection

Coverage

Connectivity

Homotopic Classes

Path Quality

Can be achieved even without storing the dense graph  
[Dobson, Bekris ICRA’13]
Properties of SPARS methods

• Achieves **probabilistic completeness** through coverage and connectivity criteria, as visibility-based PRM.

• With probability approaching 1 as consecutive failures, M, goes to infinity, SPARS2 will **cover all arbitrary optimal paths**.

• Paths in the Roadmap Spanner have **length bounded** by an input stretch factor, $t$, with probability approaching 1.

• SPARS2 grows the roadmap **with probability zero** as iterations increase to infinity.
Evaluation

Abstract environment in the Open Motion Planning Library:

Path Length vs PRM* (Abstract)

Answered Queries out of 1000 (Maze t=2)

Average Valence (Abstract T=2)

Offline Memory

ge (Abstract t=2)

Memory Usage (Bytes)
Conclusion/Future Work

Roadmap spanners are practical solutions with desirable properties for high-dim motion planning

- Available in the next release of OMPL
- Work in progress:
  - Show manipulation solutions using MoveIt
  - Study roadmaps with directed edges
  - Finite time properties of sampling-based planners

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