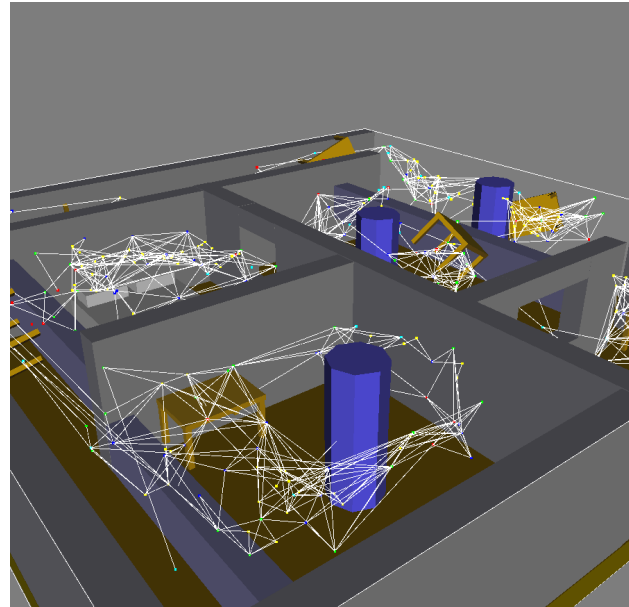




## Roadmap Spanners



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Joint work with Andrew Dobson and James Marble

Department of Computer Science

Rutgers University

May 10<sup>th</sup>, 2013

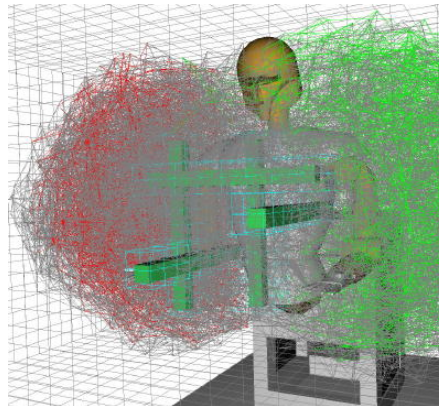
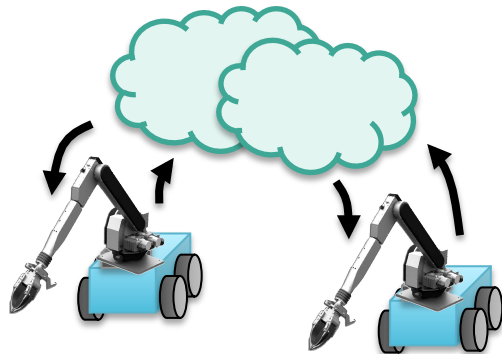


# Why Roadmaps and Roadmap Spanners?

Produce sparse graphical representations that:

- reflect the connectivity of the configuration space
- and can be used to efficiently answer online queries with good quality paths

- Posed as an important challenge for motion planning [Agarwal, '11]
- Good for resource constrained robots, potentially interfacing with a computing cloud



From the work  
on “Dynamic Roadmaps”  
by Kallmann, Mataric

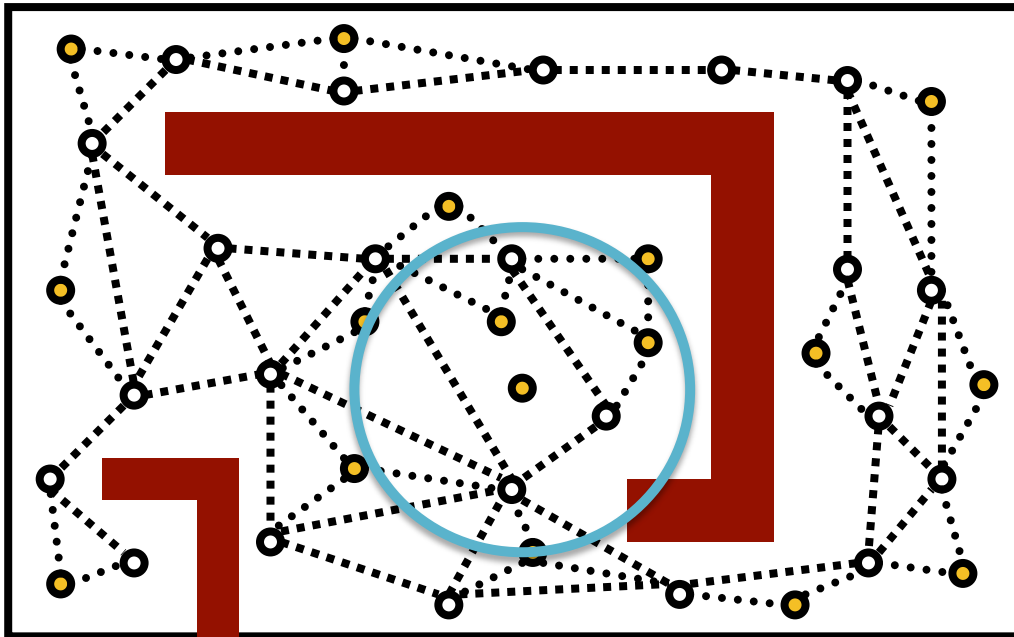
- Useful in higher-dimensional challenges such as mobile manipulation:
  - Roadmaps can store experience! They are path libraries!

NEAR-  
**OPTIMAL**  
**MOTION**



# Roadmaps and Path Quality

- A fully connected graph gives asymptotically optimal solutions
  - Resembles exhaustive search, results quickly in a huge data structure
- Connecting to  $k$  closest neighbors is efficient [PRM, Kavraki et al. '96]
  - Doesn't result in an asymptotically optimal solution for constant  $k$



## From percolation theory

It is sufficient if we attempt to connect any new sample with approximately  $k = \log n$  neighbors, where  $n$  is the number of nodes in the roadmap.

[kPRM\* - Karaman, Frazzoli '11]

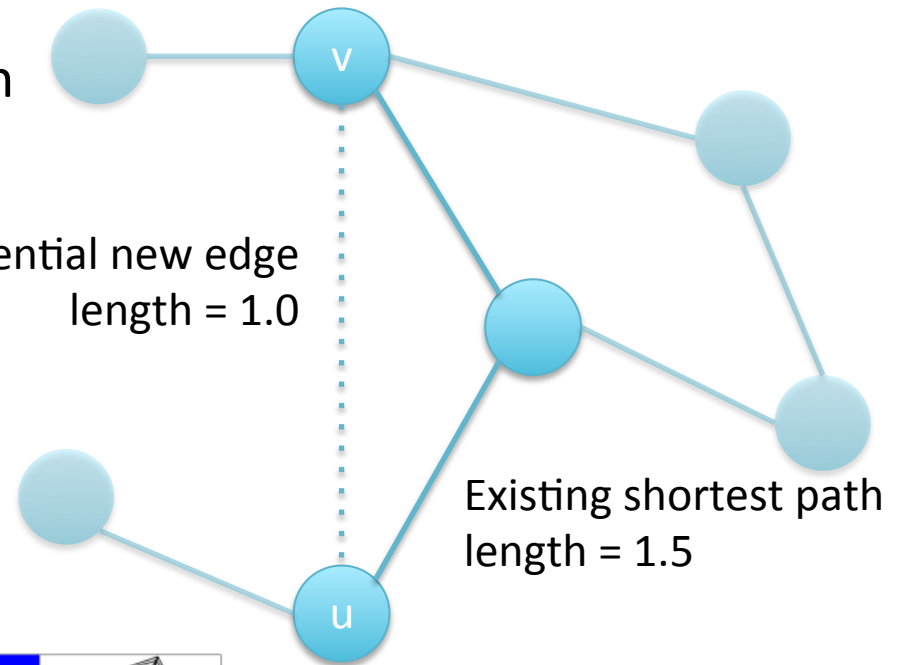
- Efficiency challenge
  - Asymptotically optimal roadmaps are large and dense



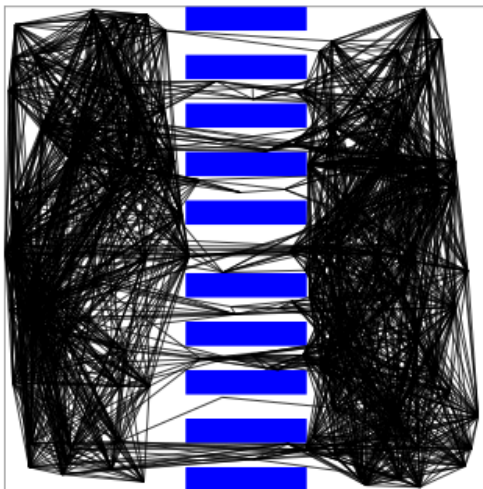
# Asymptotic Near-Optimality & Graph Spanners

- A  $t$ -spanner is a sparse subgraph
- For every shortest path in the original graph
  - There is a path in the spanner that is no longer than  $t$  times the original length

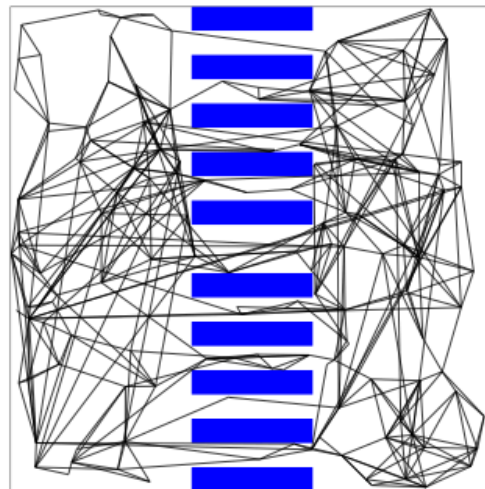
Potential new edge  
length = 1.0



Existing shortest path  
length = 1.5



(a) Roadmap with 2346 edges



(b) Spanner with 470 edges

Giving rise to a sequential approach:

- Compute  $k$ -PRM\*
- Return its spanner

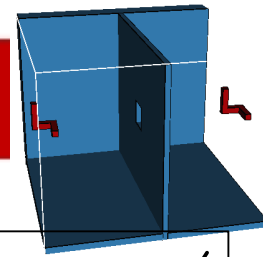
[Marble, Bekris IROS '11]

[Based on the graph spanner approach  
by Baswana, Sen '07]



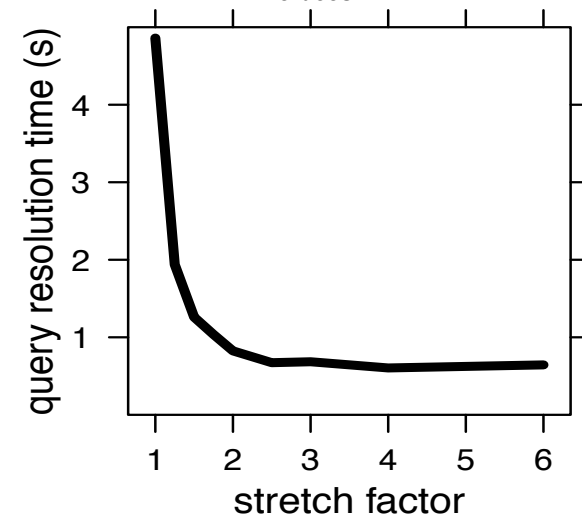
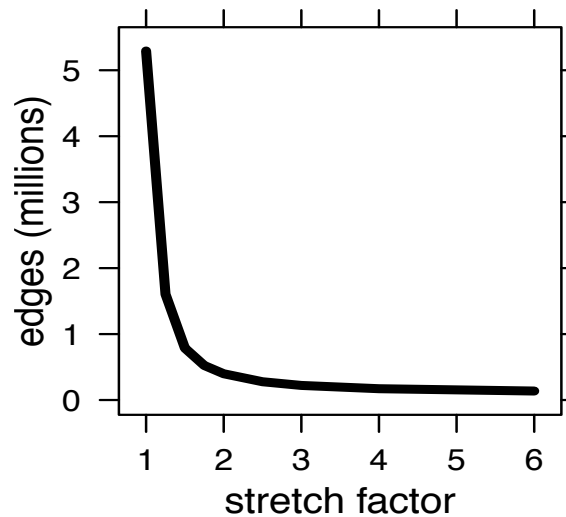
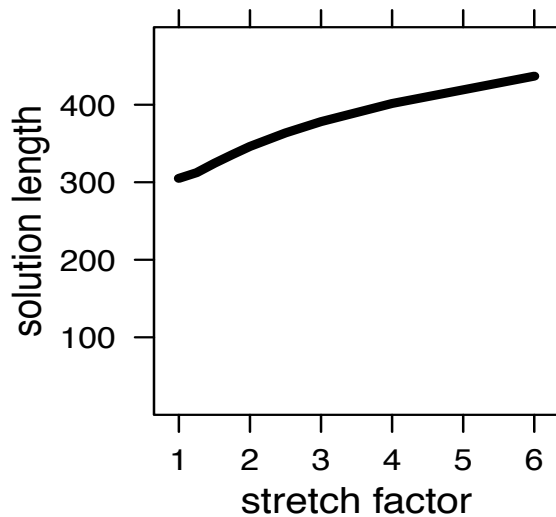
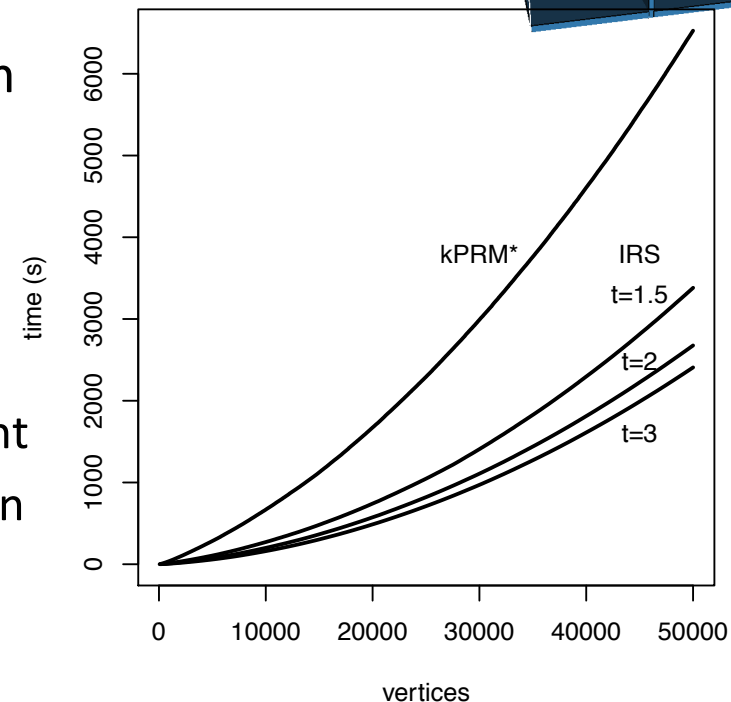
# Incremental Roadmap Spanner

Results on  
OMPL



- Start with the asymptotically optimal k-PRM\*
- Interleave an incremental spanner algorithm
- Result: An asymptotically *near*-optimal planner
  - Smaller average increase in path length than the stretch factor
  - Sparse roadmap with smaller memory footprint
  - Faster construction and online query resolution

[Marble, Bekris ISRR '11,  
IEEE Transactions on Robotics '13]





# Sparse Roadmap Spanner (SPARS)

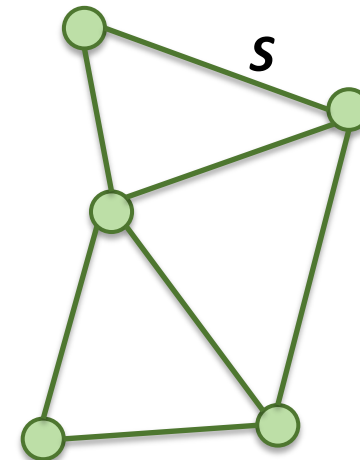
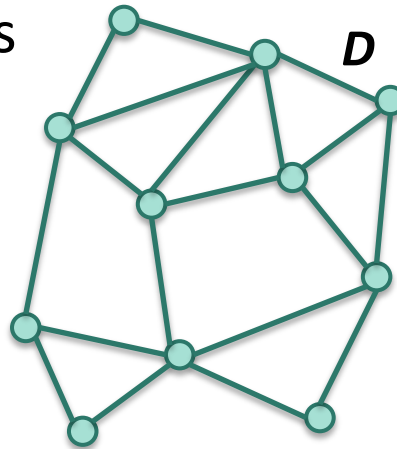
- Up to this point: Solutions add all samples in the roadmap
- Idea: Asymptotic Near-Optimality with Additive Cost

[Dobson, Krontiris, Bekris WAFR '12, IJRR '13 (accepted)]

- Consider two graphs in parallel:

Dense Graph:

- Asymptotically Optimal ( $\delta$ -PRM\*)



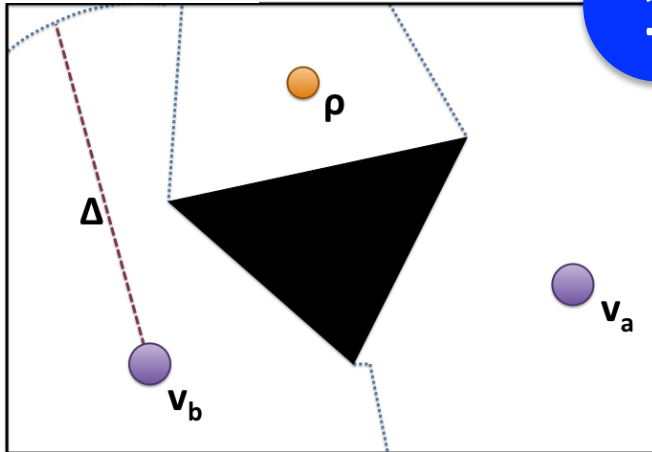
Roadmap Spanner:

- Asympt. Near-Optimal
- Not all nodes added (!)
- When should samples be added to  $S$ ?
  - If necessary for coverage, connectivity, optimality
- When should the sampling stop?
  - Criterion: After  $M$  consecutive failures to add a node



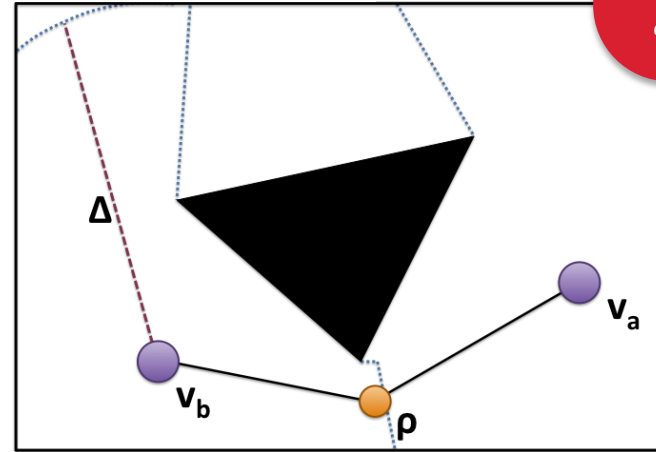
# SPARS: Node Selection

Coverage



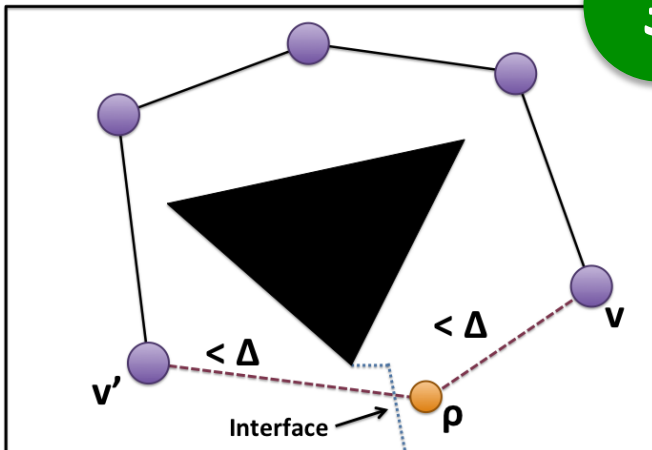
1

Connectivity



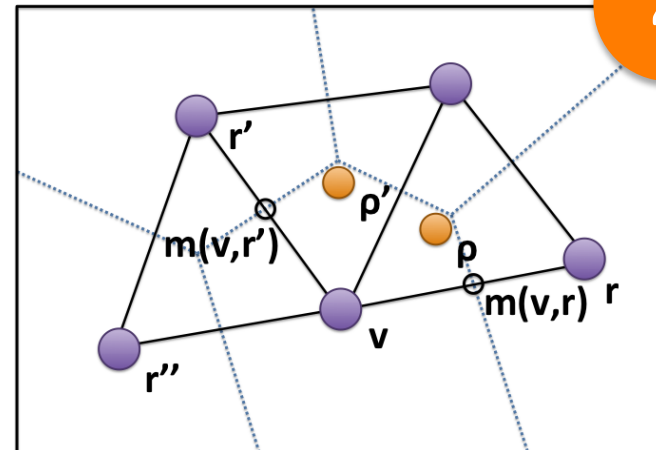
2

Homotopic Classes



3

Path Quality



4

Can be achieved even without storing the dense graph

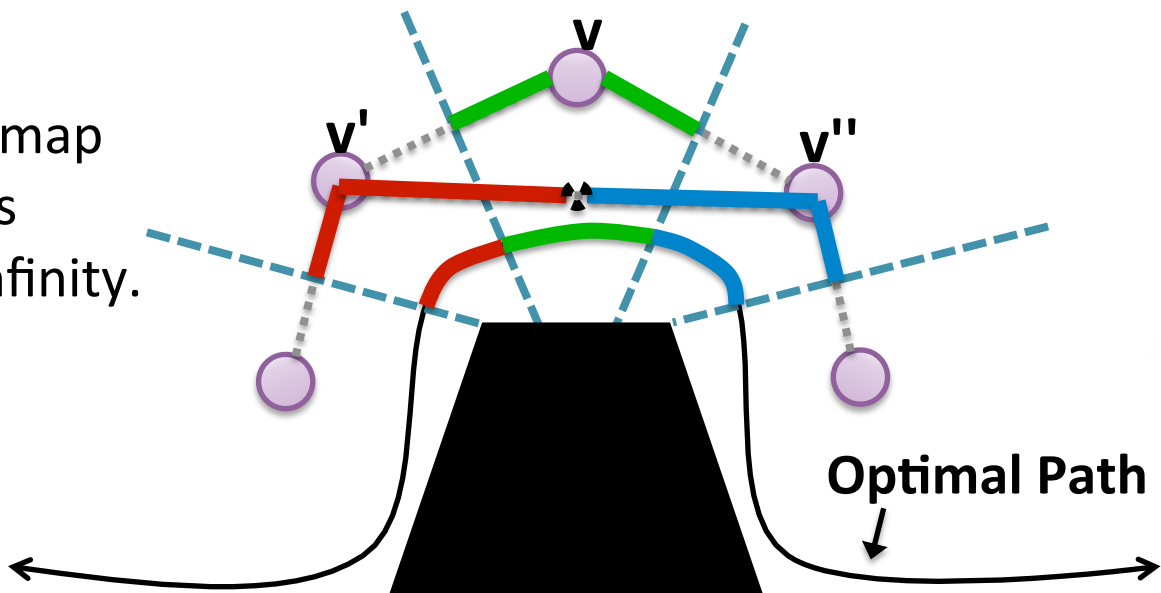
[Dobson, Bekris ICRA'13]





# Properties of SPARS methods

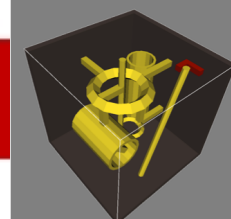
- Achieves **probabilistic completeness** through coverage and connectivity criteria, as visibility-based PRM.
- With probability approaching 1 as consecutive failures,  $M$ , goes to infinity, SPARS2 will **cover all arbitrary optimal paths**.
- Paths in the Roadmap Spanner have **length bounded** by an input stretch factor,  $t$ , with probability approaching 1.
- SPARS2 grows the roadmap **with probability zero** as iterations increase to infinity.



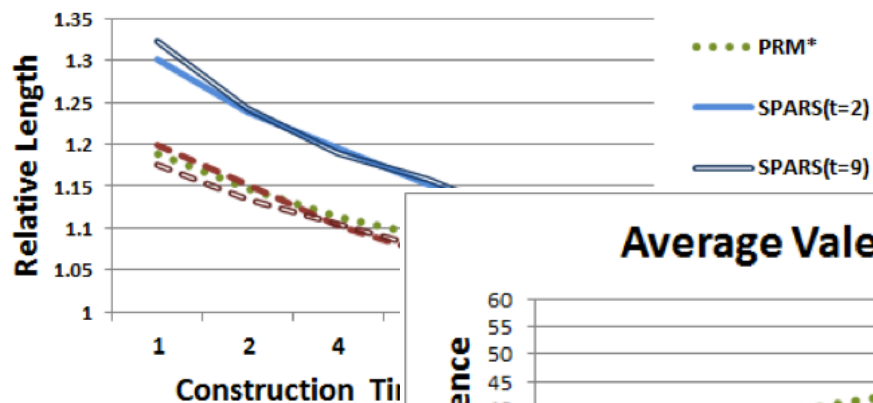


# Evaluation

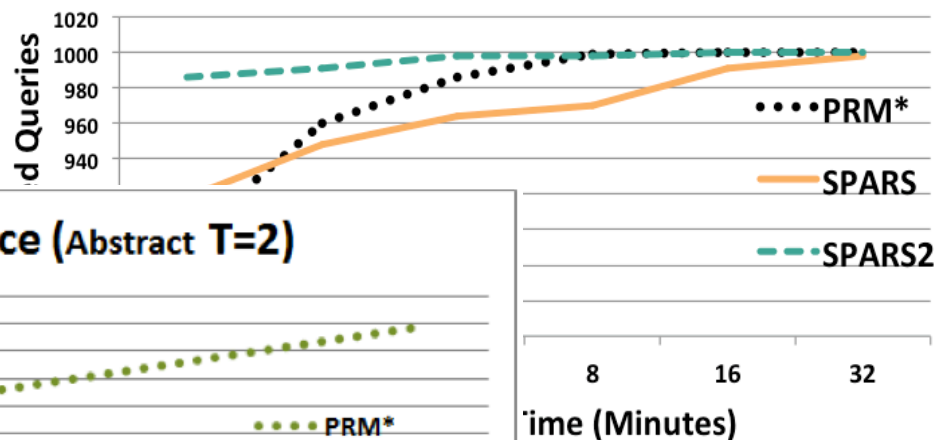
Abstract environment in the  
Open Motion Planning Library:



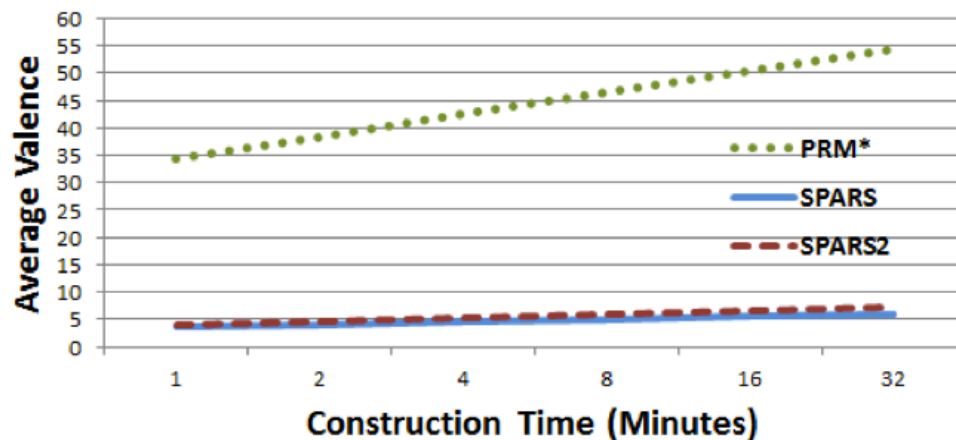
### Path Length vs PRM\* (Abstract)



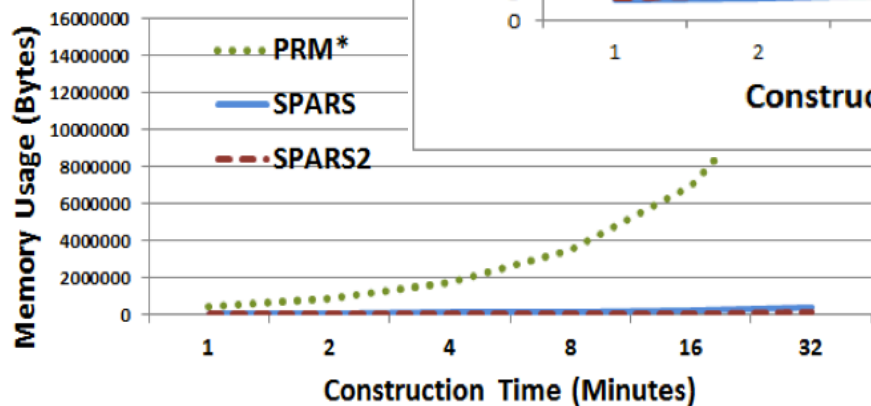
### Answered Queries out of 1000 (Maze t=2)



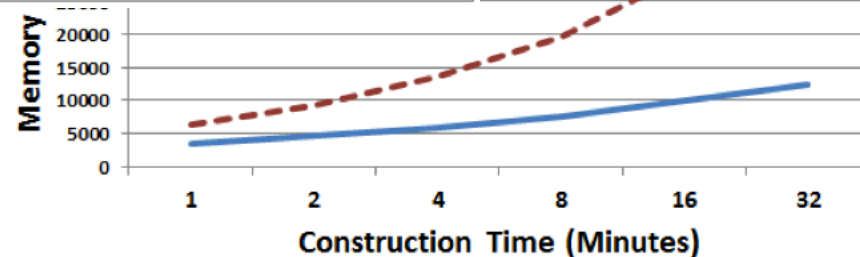
### Average Valence (Abstract T=2)



### Offline Memory



### Memory Usage (Abstract t=2)





# Conclusion/Future Work

[www.pracsyslab.org](http://www.pracsyslab.org)

Roadmap spanners are practical solutions with desirable properties for high-dim motion planning

- Available in the next release of OMPL
- Work in progress:
  - Show manipulation solutions using MoveIt
  - Study roadmaps with directed edges
  - Finite time properties of sampling-based planners



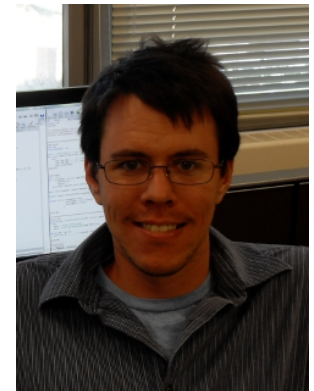
**Thank you!**

We would like to thank the **CPS** program of the **National Science Foundation** for its support

- NSF CNS 0932423



**Andrew Dobson**



**James Marble**